AIST-PS-2005-010

Kleene category as a model of calculation

(Preliminary Version)

Takeuti Izumi

AIST

Kleene category as a model of calculation

(Preliminary Version)

Takeuti Izumi

AIST

Kleene category as a model of calculation

Takeuti Izumi

takeuti@ni.aist.go.jp

Abstract: Kleene algebra is an algebraic abstraction of regular languages, and Kleene category is a generalisation of Kleene algebra. This paper shows that, for several models of calculation, their denotational semantics are given as Kleene categories.

1 Introduction

There have been many models of calculation and dynamics. An automaton is regarded as a model of symbolic dynamics as well as a recognition of a language, which is a most simple calculation. The Turing machine and the recursive function are the standard model of calculation, and the calculational powers of them are equivalent to each other. A BSS machine is not a model of calculation but a simple and curious model of algebraic dynamics with non-algebraic functions [1]. A Type-2 machine gives a model of calculation over continuous spaces [8].

Such models are given as instances of transition systems where the transitions are relatoins between states [4]. If the transition system has the structure of category, that is, it has the composition of transitoins, then it has all the fninite traces of calculaiton.

Kleene algebra is a algebraic abstraction of regular languages [6], and Kleene category is a generalisation of Kleene algebra [5]. A Kleene algebra gives the denotational semantics of a programming language [2]. This paper shows that Kleene categories give the denotational semantics of the models above. We define an operation called Kleene completion so that The Kleene completion of a model of calculation gives its denotational semantics.

In Section 2, we give the definition of transition system and also give Kleene completion which makes Kleene categories from transition systems. Section 3 shows labelled transition systems and regular languages. In Section 4, We define a category of partial functions. As instances of categories of partial functions, we show Recursive functions in section 5, and BSS machine in Section 6. Finally we show type-2 machines and the semantics as the Kleene completion in Section 7.

2 Transition system, category and Kleene category

Definition 2.1 (Kleene category) A triple $(C, +, *)$ is a *Kleene category* iff:

1. C is a category.
2. Each hom-set is idempotent commutative monoid with the monoidal operation $+$. We write $f \leq g$ iff $f + g = g$.

3. The operation $\circ$, the composition of arrows, is distributive that is, $(f + f') \circ g = f \circ g + f' \circ g$ and $g \circ (f + f') = g \circ f + g \circ f'$

4. For each object X, there is the unary operation * over $Hom(X, X)$ such that, for each arrow f in $Hom(X, X)$, the followings hold:

   (a) $f^* = 1 + f \circ f^* = 1 + f^* \circ f$, where 1 is the identity arrow.

   (b) $h + f \circ g \leq g$ implies $h + f^* \leq g$, and $h + g \circ f \leq g$ implies $h + f^* \leq g$.

Remark 2.2 This definition of Kleene category appears in the literature [5].

Definition 2.3 (Transition system) A pair $S = (Q, T)$ is a *transition system* if:

1. Q is a finite set.

2. $T(q, q')$ is a set for any $q, q' \in Q$.

The elements of Q are called states. We call an element of T a transition. If f in T(q,q'), then we say that one transits from q to q' by a transition f.

Definition 2.4 (Subsystem) A transition system $S = (Q, T)$ is a *subsystem* of a transition system $S'(Q', T')$ if $Q \subset Q'$ and $T(q, q') \subset T'(q, q')$ for any $q, q' \in Q$.

Remark 2.5 A category is regarded as a transition system S with the objects as the states and the arrows as the transitions.

Definition 2.6 (Compositional completion) Let a transitoin system $S_0 = (Q, T_0)$ be a category and $S = (Q, T)$ be a subsystem of S_0. Then, the *compositional completion* of S in S_0 is the smallest category S' which is a subsystem of S_0 and includes S. We write $C(S)$ for the compositional completion of S.

Definition 2.7 (Kleene completion) Let a transitoin system $S_0 = (Q, T_0)$ be a Kleene category and $S = (Q, T)$ be a subsystem of S_0. Then, the *Kleene completion* of S in S_0 is the smallest Kleene category S' which is a subsystem of S_0 and includes S. We write $K(S)$ for the Kleene completion of S.

3 Labelled transition system

Notation 3.1 We put Σ as a finite set of characters and fix it.

We write Σ^* for the set of all the finite sequences of the elements of Sigma. We call its elements words. The set Σ^* has the empty word ϵ.

The set $Lang(\Sigma)$ is defined as the power set of Σ^*. An element of $Lang(\Sigma)$ is called a language over Σ. For $X, Y \in Lang(\Sigma)$, we write $X \cdot Y$ for $\{vw | v \in X \& w \in Y\}$. For $X \in Lang(\Sigma)$, we write X^* for $\{w_1 w_2 ... w_n | w_i \in X\}$, which is socalled the Kleene star.

Notation 3.2 $T_{Q,\Sigma}$ is the function such that $T_{Q,\Sigma}(q, q') = \{q\} \times \Sigma \times \{q'\}$.

Definition 3.3 (Labelled transition system) A transition system (Q, T) is a labelled transition system with Σ iff $T(q, q') \subset T_{Q,\Sigma}$ for any $q, q' \in Q$. The elements of Σ are called characters. If $(q, \sigma, q') \in T$, then we say that one can transit from s to s' by σ.

Definition 3.4 (Determinism) A labelled transition system $M = (Q, T)$ is *deterministic* if, for any $q \in Q$ and $\sigma \in \Sigma$, there is at most one q' such that $(q, \sigma, q') \in T(q, q')$.

Remark 3.5 A labelled transition system (Q, T_{Q,Σ^*}) has the structure of categorey with the identity (q, ϵ, q) and the composition of arrows $(q', w, q'') \circ (q, v, q') = (q, vw, q'')$.

A character set Σ is regarded as a subset of Σ^* with a character is identified as the one-letter word. For a labelled transition system $M = (Q, T)$ with the characoter set Σ, the set of its transitions $T(q, q')$ is a subset of $Q \times \Sigma^* \times Q$. Therefore, we have $C(M) = (S, CT)$ which is the compositional completion of M in the transition system (Q, T_{Q,Σ^*}).

Lemma 3.6 *A labelled transition system is deterministic iff the compositional completion is deterministic.*

Remark 3.7 A labelled transition system $(Q, T = Q, Lang(\Sigma)$ is Kleene completed with the following data:

- $(q, \{\epsilon\}, q)$ as the identity,
- $(q', Y, q'') \circ (q, X, q') = (q, X \ldots Y, q'')$ as the composition of arrows,
- $(q, X, q') + (q, Y, q') = (q, X \cup Y, q')$ as the monoidal operation,
- $(q, \emptyset, q')$ as the unit of monoid,
- $(q, X, q')^* = (q, X^*, q')$ as the unary operation.

Remark 3.8 Let $M = (Q, T)$ be a labelled transition system. We have another labelled transition system $\hat{M} = (Q, \hat{T})$ where $\hat{T}(q, q') = \{(q, \sigma, q') | (q, \sigma, q') \in T(q, q')\}$. This $\hat{M}$ is isomorphic to M, and is subsystem of $(Q, T_{Q,Lang(\Sigma)})$. Therefore, we have $K(\hat{M}) = (Q, KT)$ which is the compositional completion of $\hat{M}$ in the Kleene catregory $(Q, T_{Q,Lang(\Sigma)})$. We abbreviate $K(\hat{M})$ as $K(M)$ and say simply the Kleene completion of M.

Lemma 3.9 *A labelled transition system $M = (Q, T)$ is deterministic iff the Kleene completion $K(M) = (Q, KT)$ satisfies the condition that, for any q, q', q'' $\in Q$, if $q' \neq q''$ then $KT(q, q') \cap KT(q, q'') = \emptyset$.*

Theorem 3.10 *Let $M = (Q, T)$ be a labeled transition system where S and Σ are finite sets. Let CT be the set such that $C(M) = (Q, CT)$ and KT be the set such that $K(M) = (Q, KT)$.*

Put $q, q' \in Q$, and X and $\hat{X}$ as the sets such that $X = \{w | (q, w, q') \in CT(q, q')\}$ and $(q, \hat{X}, q') = KT(q, q')$. Then:

1. *Each element of $\hat{X}$ is regular.*
2. *The language X is the largest element of $\hat{X}$.*

4 Relations

Definition 4.1 (Kleene Category of Relations) For sets X and Y, the notation $Rel(X, Y)$ denotes the power set of $X, \times Y$. Let Q be a set each element of which is a set. We write $Rel(Q)$ for The Kleene category of relations over Q which is defined as a transition system $Rel(Q) = (Q, T)$ where $T(X, Y) = Rel(X, Y)$ with the following operations:

1. $f \cup g$ of f and g in $Rel(X, Y)$ as the monoidal operation,
2. $g \circ f = \{(x, z) \in X \times Z | (x, y) \in f, (y, z) \in g\}$ of $f \in Rel(X, Y)$ and $g \in Rel(Y, Z)$ as the composition,
3. $f^* = \{(x, y) \in X \times X|$
   For some $n \geq 0$ and $(x_0, x_1, x_2, ..., x_n) \in X^{n+1}$, $x = x_0$, $y = x_n$,
   $(x_{i-1}, x_i) \in f$ for each $i = 1, 2, ..., n$.
   $\}$ of $f \in Rel(X, X)$ as the unary operation.

Definition 4.2 (Partial function) A function f is a *partial function* of X into Y if f is a function of X' into Y where X' is a subset of X. The domain of partial function is written as $dom(f)$.

We identify a partial function f of X to Y to the relation $\{(x, f(x)) | x \in dom(f)\} \in Rel(X, Y)$.

Definition 4.3 (Deterministic machine) Let $S = (Q, T)$ be a transition system which is a subsystem of $Rel(Q)$. This S is a *deterministic machine* if:

1. Each $f \in T(X, Y)$ is a partial function.
2. $dom(f) \cap dom(g) = \emptyset$ for any $f \in T(X, Y)$ and $g \in T(X, Z)$.
3. There is a state $q_F \in Q$ such that , for any $q \in Q$, the set $T(q_F, q)$ is the empty set or the singleton $\emptyset$.

This q_F is called a *final state.*

Theorem 4.4 *Let $S = (Q, T)$ be a deterministic machine with a final state $q_F \in Q$. Take $K(S) = (Q, KT)$ as the Kleene completion of S in $Rel(Q)$. Then the followings hold:*

1. *Each $f \in KT(q, q_F)$ is a partial function.*
2. *$KT(q, q_F)$ is a singleton $\{f\}$ of some partial function f if it is not empty.*

5 Recursive function

Definition 5.1 (Modified Turing machine) A transition system $S = (Q, T)$, which is a subsystem of $Rel(Q)$, is called a *Modified Turing machine* if:

1. Q is a finite family of sets such that each $q \in Q$ is a copy of $\mathbf{N}^{n_q}$, where n_q depends on q. We regard $q \neq q'$ in the element of Q even if $n_q = n_{q'}$.

2. Each $f \in T(q, q')$ is a partial recursive function of q into q'.

3. S is a deterministic machine.

Definition 5.2 (Turing computable) A partial function f of $\mathbf{N}^m$ into $\mathbf{N}^n$ is *Turing computable* if there is a Modified Turing machine $S = (Q, T)$ where its Kleene completion is $K(S) = (Q, KT)$ and its final state is $q_F \in Q$ such that:

1. q is isomorphic to $\mathbf{N}^m$ and q_F is isomorphic to $\mathbf{N}^n$.

2. Put i as the isomorphism of $\mathbf{N}^m$ into q', and j as the isomorphism of q_F into $\mathbf{N}^n$. Then $i' \circ f \circ i$ is the maximum of $KT(q, q_F)$.

Definition 5.3 (Category of recursively enumerable relations) Let Q be a family of sets as above. The *category of recursively enumerable relations* over Q is a transition system (Q, T) such that $T(q, q')$ consists of all the recursively enumerable sets in *Rel*(q, q'). This transition system is Kleene completed in *Rel*(Q).

Remark 5.4 Put the category of recursively enumerable relations (Q, T). Let e be a recursively enumerable subset of $q \in Q$. Then a diagonal relation $\{(x, x) | x \in e\} \in T(q, q)$ is regarded as a test, although the set of these diagonals does not forms a Boolean algebra. Thus, we can regard this as a Kleene category with an embedded structure, which is an analogy of a Kleene algebra with an embedded structure [3].

Remark 5.5 Let Q as above, and q, q' be elements in Q. For a partial function f of q into q', the following three are equivalent: 1. f is a partial recursive function. 2. f is a partial function and is a recursively enumerable set in *Rel*(q, q'). 3. f is Turing computable.

6 BSS machine

Remark 6.1 We put a field K and fixed it. The typical example is $\mathbf{R}$, the filed of all the real numbers.

Definition 6.2 (Variety) A *variety* is a set

$$\{(x1, x2, ..., xn) \in K^n | \\ p_1(x_1, x_2, ..., x_n) = 0, p_2(x_1, x_2, ..., x_n) = 0, ..., p_m(x_1, x_2, ..., x_n) = 0$$

where p_1, p_2, ..., p_m are polynomials in K.

Remark 6.3 Note that we have infinitely many varieties which are isomorphic to each others. Actually, the variety $\{x \in K^n | p(x) = 1\}$ is distinct from $\{(x, x_{n+1}) \in K^{n+1} | p(x) = 1, x_{n+1} = 0\}$ but isomorphic to it.

Definition 6.4 (Rational function) Let $M \subset K^m$ and $N \subset K^n$ be varieties. A partial function f of M into N is *rational* if there are polynomials

$$p_1(x_1, x_2, ..., x_m), p_2(x_1, x_2, ..., x_m), ..., p_n(x_1, x_2, ..., x_m),$$
$$q_1(x_1, x_2, ..., x_m), q_2(x_1, x_2, ..., x_m), ..., q_n(x_1, x_2, ..., x_m)$$

such that

– $dom(f) = \{(x_1, ..., x_m) \in M | p_i(x_1, x_2, ..., x_m) \neq 0$ for each $i = 1, 2, ..., n\}$,

– $(y_1, y_2, ..., y_n) = f(x_1, x_2, ..., x_m)$ iff $y_i = q_i(x_1, x_2, ..., x_m)/p_i(x_1, x_2, ..., x_m)$ for each i.

Definition 6.5 (Fundamental test) We put a finite partition F over K, that is, this F is a finite subset of the power set of K and satisfies that, for each $x \in K$, there is a unique $e \in F$ such that $x \in e$. We assume that $\{0\} \in F$. We fix this F in this section and we call an element of F a *fundamental test*. The typical example is $\{\{0\}, \{x \in \mathbf{R} | x > 0\}, \{x \in \mathbf{R} | x < 0\}\}$ over $\mathbf{R}$.

Definition 6.6 (Atomic test) Let M be a variety. A set $e \subset M$ is called an *atomic test* if there is a fundamental test $e' \in F$ such that $e = \{(x_1, x_2, ..., x_n) \in M | x_i \in e'\}$ or $e = \{(x_1, x_2, ..., x_n) \in M | x_i \notin e'\}$ for some i. For an atomic test $e \subset M$, its diagonal $\{(x, x) \in M \times M | x \in e\} \in Rel(M, M)$ is also called an atomic test.

Remark 6.7 This word of test is in the sense of test in Kozen's Kleene category with test [7].

Definition 6.8 (BSS machine) A transition system $S = (Q, T)$, which is a subsystem of $Rel(Q)$, is called a *BSS machine* if:

1. Each $M \in Q$ is a variety.
2. Each $f \in T(M, N)$ is either g or $g \circ h$ for an rational function $g \in Rel(M, N)$ and an atomic test $h \in Rel(M, M)$.
3. S is a deterministic machine.

Definition 6.9 (BSS-computable) Let M and N be varieties. A partial function f of M into N is *BSS-computable* if there is a BSS machine $S = (Q, T)$ where its Kleene completion is $K(S) = (Q, KT)$ and its final state is $N' \in Q$ such that:

1. N' is isomorphic to N.
2. Put i be the isomorphism of N into N'. Then $i \circ f$ is the maximum of $KT(M, N')$.

Remark 6.10 We do not suppose $N = N' \in Q$ in fear that $M = N$.

Remark 6.11 The original BSS machine is a BSS machine $S = (Q, T)$ of the definition above which satisfies the followings:

1. Q consists of some copies of $\mathbf{R}^n$ for some fixed n.

2. The fundamental test is $F = \{\{0\}, \{x \in \mathbf{R} | x > 0\}, \{x \in \mathbf{R} | x < 0\}\}$.

3. Each $f \in T(M, N)$ is one of the followings:

   (a) $dom(f) = \{(x_1, x_2, ..., x_n) | x_i \in e\}$ for some $e \in F$, and $f(x) = x$.

   (b) $dom(f) = \{(x_1, x_2, ..., x_n) | x_i \notin e\}$ for some $e \in F$, and $f(x) = x$.

   (c) $dom(f) = \mathbf{R}^n$,
   $f(x_1, x_2, ..., x_n) = (x_1, ..., x_{i-1}, p(x_1, x_2, ..., x_n), x_{i+1}, ..., x_n)$ for some polynomial p with integer coefficients.

   (d) $dom(f) = \{(x_1, x_2, ..., x_n) \in \mathbf{R}^n | x_j \neq 0\}$,
   $f(x_1, x_2, ..., x_n) = (x_1, ..., x_{i-1}, x_k/x_j, x_{i+1}, ..., x_n)$

Definition 6.12 (BSS category) Let Q be a family of varieties. The *BSS category* over Q is defined as follows. Let $S = (Q, T)$ is a transition system such that:

1. $T(M, M)$ consists of all the rational functions and all the atomic tests over M.

2. $T(M, N)$ consists of all the rational functions of M into N, for $M \neq N$.

The BSS category over Q is a Kleene completion of S in *Rel*(Q).

Theorem 6.13 *Let $S = (Q, T)$ be a BSS category over Q. Put $M, N \in Q$. Then, for a partial function f of M into N, it holds that f is BSS computable iff $f \in T(M, N)$.*

Remark 6.14 A calculation of a BSS machines is a composition of the operations of the field. It is not a model of computation of real numbers as a topological set.

7 Type-2 machine

Notation 7.1 We take two disjoint finite sets Σ and Σ'. We regard elements of Σ as input characters, elements of Σ' as output characters.

Definition 7.2 (Type-2 machine) A transition system $M = (Q, T)$ is a *type-2 machine* with the input character set Σ and the output chaarcter set Σ' if:

1. Q is finite.

2. All $q \in Q$ are disjoint to each other, and isomorphic to $\mathbf{N}$.

3. Each $f \in T(q, q')$ is a recursively enumerable subset of $Q \times (\Sigma \cup \Sigma' \cup \{\epsilon\}) \times Q$.

Definition 7.3 (Determinism) A type-2 machine $M = (Q, T)$ is *deterministic* if:

1. For each $q \in Q$ and each $x \in q$, at most one of the followings hold:

(a) $(x, \sigma, y) \in T(q, q')$ for some $\sigma \in \Sigma$, $q' \in Q$, and $y \in q'$.

(b) $(x, \tau, y) \in T(q, q')$ for some $\tau \in \Sigma' \cup \{\epsilon\}$, $q' \in Q$, and $y \in q'$.

2. For each $q \in Q$, each $x \in q$ and each $\sigma \in \Sigma$, there are at most one $q' \in Q$ and at most one $y \in q'$ such that $(x, \sigma, y) \in T(q, q')$.

3. For each $q \in Q$ and each $x \in q$, there are at most one $q' \in Q$, at most one $y \in q'$ and at most one $\tau \in \Sigma' \cup \{\epsilon\}$, such that $(x, \epsilon, \tau, y) \in T(q, q')$.

Remark 7.4 One can regard a type-2 machine $MA = (Q, T)$ as a labelled transition system which is a subsystem of $(\bigcup Q, T_{Q,\Sigma\cup\Sigma'\cup\{\epsilon\}})$. If M is a deterministic type-2 machine, then it is a deterministic labelled transition system. The inverse does not hold, because a deterministic labelled transition system may have (x, σ, y) and (x, τ, z) where both *sigma* and τ belong to $\Sigma' \cup \{\epsilon\}$ and *sigma* $\neq \tau$.

In case $\sigma' = \emptyset$, then that never happens. Therefore, if $\sigma' = \emptyset$, then M is a deterministic type-2 machine iff it is a deterministic labelled transition system with the character set $\Sigma \cup \{\epsilon\}$.

Definition 7.5 (Full Kleene category) The *full Kleene category* over (Q, Σ, Σ') is a transition system $S_0 = (Q, T_0)$ defined as:

1. $T_0(q, q')$ is the power set of $Q \times \Sigma^* \times \Sigma'^* \times Q$.

2. $g \circ f = \{(x, v \cdot w, v' \cdot w', z) | (x, v, v', y) \in f, (y, w, w', z) \in g\}$ as the composition of arrow.

3. $\{(x, \epsilon, \epsilon, x) | x \in q\}$ as the identity over $q \in Q$.

4. $\cup$ is the monoidal operation with the unit $\emptyset$.

5. $f^* = \{(x, w_1 \cdot w_2 \cdot \ldots \cdot w_n, w'_1 \cdot w'_2 \cdot \ldots \cdot w'_n, y) |$
   $x = x_0,\ y = x_n,\ (x_{i-1}, w_i, w'_i, x_i) \in f$ for $i = 1, 2, \ldots, n$
   $\}$ as the unary operation.

Definition 7.6 (Determinism of transitions) For a transition $f \subset Q \times \Sigma \times \Sigma^* \times Q$ is deterministic if the following holds.

1. Put $q, r, s \in Q$, $x \in q$, $y \in r$, $z \in s$, $v \in \Sigma^*$ and $w, w' \in \Sigma'^*$. If $(x, v, w, y) \in T(q, r)$ and $(x, v, w', z) \in T(q, s)$, then either w is a prefix of w' or w' is a prefix of w.

2. Put $q, r, s \in Q$, $x \in q$, $y \in r$, $z \in s$, $v, v' \in \Sigma^*$ and $w, w' \in \Sigma'^*$. If $(x, v, w, y) \in T(q, r)$, $(x, v', w', z) \in T(q, s)$, and v is the proper prefix of v', then either w is a prefix of w'.

Remark 7.7 Kleene completion

For a type-2 machine $M = (Q, \Sigma, \Sigma', T)$, the transition system $\hat{M} = (Q, \hat{T})$ whish is a subsystem of the full kleene category over (Q, Σ, Σ') is constructed as:

$$
\begin{aligned}
&\hat{T}(q, q') = \{ \\
&\quad \{(q, \sigma, \epsilon, q') | \sigma \in \Sigma, (q, \sigma, q') \in T(q, q')\} \\
&\quad \cup \{(q, \epsilon, \tau, q') | \tau \in \Sigma', (q, \tau, q') \in T(q, q')\} \\
&\quad \cup \{(q, \epsilon, \epsilon, q') | (q, \epsilon, q') \in T(q, q')\} \\
&\}
\end{aligned}
$$

There is $K(\hat{M})$ the Kleene completion of $\hat{M}$. We write $K(M)$ for this $K(\hat{M})$ and call it the *Kleene completion of the type-2 machine M*.

Theorem 7.8 *Let $M = (Q, T)$ a type-2 machine. Put $K(M)$ the Kleene completion of M in the full Kleene category. Then, each transition f of $K(M)$ is deterministic and a recursively enumerable subset of $Q \times \Sigma^* \times \Sigma'^* \times Q$.*

8 Future work

We have shown that Kleene completion gives the semantics for several models of calculation and dynamics. This paper discuss only finite calculation, although a type-2 machine is regarded as a model of infinite calculation. One requires more studies of the semantics of the model of infinite calculation as the future work.

References

[1] Blum, L., Shub, M. & Smale, S.: 'On a Theory of Computation Over the Real Numbers; NP Completeness, Recursive Functions and Universal Machines', Bulletin of the AMS, Vol.21, No.1, pp1–46, 1989.

[2] Furusawa, H & Kinoshita, Y.: 'Essentially algebraic structure for Kleene algebra with tests and its application to semantics fo while program', Transaction of Information Processing Society of Japan, Vol.44, No. SIG 4, pp47–53, 2002.

[3] Furusawa, H.: 'A framework for Kleene algebra with an embedded structure', Participants proceedings of 8th International Conference on Relational Methods in Computer Science, pp77–84, 2004.

[4] Kawahara, Y.: 'Apploication of relational calculus to computer mathematics', Bulletin of Informatics and Cybernetics, Vol.23, pp67–78, 1998.

[5] Kinoshita, Y.: 'Algebraic structures around fixed points', Proceedings of 18th Annual Conference of Japan Society for Software Science and Technology, 2001.

[6] Kozen, D.: ‘A Completeness Theorem for Kleene Algebras and the Algebra of Regular Events’, Information and Computation, Vol.110, No.2, pp366–390, 1994.

[7] Kozen, D.: ’Kleene Algebra with Tests’, ACM Transaction on Programming Language and Systems, Vol.19 No.3 pp427–443, 1997.

[8] Weihrauch, K.: Computable analysis: an introduction, Springer-Verlag, 2000.

計算模型としてのクリーニ圏 (in English)
(算譜科学研究速報)
発行日：2005 年 6 月 3 日
編集・発行：独立行政法人産業技術総合研究所
システム検証研究センター
同連絡先：〒661-0974 兵庫県尼崎市若王寺 3-11-46
e-mail：informatics-inquiry@m.aist.go.jp

Kleene category as a model of calculation
(Programming Science Technical Report)
3 June 2005
Research Center for Verification and Semantics (CVS)
Amagasaki Site
National Institute of Advanced Industrial Science and Technology (AIST)
3-11-46 Nakouji, Amagasaki, Hyogo, 661-0974, Japan
e-mail: informatics- inquiry@m.aist.go.jp